MATHEMATICS

MATHEMATICS

Class XII

General and subject-specific guidelines

1. The course content for Class XII will be the same as prescribed in the core syllabus in Mathematics (sr. secondary stage) brought out by the Council of Boards of school Education in India. (COBSE) in collaboration with NCERT.

2. The individual boards are free to change the weightage as per their local-specific need and requirement. It is suggested that variation may not exceed 10 percent.

3. The primary purpose of the board examination is to find out what and how much student know, not what and how much they don't know.

4. For designing a good question paper, a paper setter should combine his/her knowledge of the subject with an adequate understanding of the techniques of paper setting judiciously.

Subject-specific

5. The language of the question paper should be simple and clear, so that every question carries the same meaning for all students. Its translated version must convey the same meaning as the English version.
   S.I. units and standard mathematical symbols used in the textbook should be used.

6. Question paper must cover the entire syllabus, as shown in the blue print and design, but may be beyond recommended/prescribed books.

7. In multiple choice questions, all the options should be so designed so that guess work can be minimized.

8. Questions of the type true-false/fill in the blanks/ matching type should be avoided.

9. Questions involving long calculations requiring calculators should be avoided.

10. The question paper should be so designed that an average student must be able to complete it in the given time.

Subject: Mathematics
Maximum Marks: 100

Class: XII
Time: 3hrs

Maximum Marks:100

Weightage to assessment of objectives

Objective	Marks	Percentage
Knowledge	30	30%
Understanding	40	40%
Application	24	24%
Skill(Drawing of sketches)	06	6%

Subject: Mathematics **Class: XII**

Maximum Marks: 100 **Time: 3hrs**

Maximum Marks:100

Weightage to forms/types of questions

Form/type of questions	**Marks for each question**	**Total number of questions**	**Total Marks**
Long answer type	30%	5	30
Short answer type	52%	13	52
Very short answer type	12%	6	12
Objective type(MCQ)	6%	6	6
Total	100%	30	100

Subject: Mathematics
Maximum Marks: 100

Class: XII
Time: 3hrs

Maximum Marks:100

Weightage to difficulty level of questions

Estimated level	Marks	Percentage of marks
Difficult	20	20
Average	50	50
Easy	30	30
Total	100	100

Subject: Mathematics

Class: XII

Total Marks: 100

Time: 03

DESIGN

Unit-wise time and marks distribution

UNIT	CHAPTERS	EXPECTED PERIODS	NUMBER OF QUESTIONS	MARKS ALLOTED	TIME (IN MINUTES)
I	• Relations and Functions • Inverse trigo-nometric func-tions	10 =18 8	SA2 MCQ2	8 2 =10	15
II	• Matrices • Determinants	14 10 =24	LA1 SA1 VSA1 MCQ1	6 4 2 1 =13	20
III	• Continuity and differentiability • Application of derivatives • Integration • Application of integration • Differential eq-uations	16 16 =80 18 12 18	LA2 SA6 VSA3 MCQ2	12 24 =44 6 2	70
IV	• Vectors • 3-dimensional geometry	12 18 =30	LA1 SA2 VSA1 MCQ1	6 8 2 1 =17	25
V	• Linear pro-gramming	10=10	LA1	6=6	10
VI	• Probability	18=18	SA2 VSA1	8 2 =10	15
		180	30	100	165+15 for reading and revision

SAMPLE QUESTION PAPER

CLASS: XII

MATHEMATICS

BLUE PRINT

Assessment objectives and Distribution of forms of Questions

	Unit	Knowledge				Understanding				Application				Skills				
		LA	SA	VSA	MCQ	LA	SA	VSA	MCQ	LA	SA	VSA	MCQ	LA	SA	VSA	MCQ	
Relations and Functions	I		1		2		1											10
Matrices and Determinants	II	1			1		1	1										13
Calculus: Differentiation Integration & Diff. Eqns.	III	1	1	2	2		2	1		1	3							44
Vectors & 3-Dim Geometry	IV				1	1	1				1	1						17
Linear Programming Problems	V													1				6
Probability	VI						2	1										10
Marks		12	8	4	6	6	28	6		6	16	2		6				
Total Marks		30				40				24				6				100

- The sample paper has been prepared on the basis of above blueprint
- Having the same design, different blue prints based question papers can be developed.

SAMPLE QUESTION PAPER
CLASS: XII
MATHEMATICS

General Instructions:

Time Allowed:03Hours **Maximum Marks:100**

1. All questions are compulsory
2. The question paper consists of 30 questions divided into four sections A,B,C and D
3. Section A contains 6 questions of 1 mark each, which are multiple choice types of questions. Section B contains 6 questions of 2 marks each, Section C contains 13 questions of 4 marks each and section D contains 5 questions of 6 marks each.
4. There is no overall choice in the paper. However, internal choice is provided in one question of 2 marks, 3 questions of 4 marks each and one question of 6 marks. In questions with choices, only one of the choices is to be attempted.
5. Use of calculators is not permitted.

Section-A

Question numbers 1 to 6 carry 1 mark each. In each question, four options are provided, out which only one is correct. Select the correct option.

1. If f:R→ R, is such that fof(x)=x

   Then f^{-1} is

   (A) $\frac{1}{f}$ (B)Not defined (c) f (D)2f

2. If $\tan^{-1} x = y, then$

   (A) $0 \le y \le \pi$ (B) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ (C) $0 < y < \pi$ (D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

3. If A= $\begin{bmatrix} cosx & -sinx \\ sinx & cosx \end{bmatrix}$, and $A = A'$ then the value of x is

   $(A)\ 0 \quad (B)\frac{\pi}{2} \quad (C)\frac{\pi}{3} \quad (D)\ \frac{2\pi}{3}$

4. If f(x)= $\begin{cases} kx+1, & x \le 5 \\ 3x-4, & x > 5 \end{cases}$ is continuous at x=5, then the value k is

   (A) 4 (B) 12/5 (C) 2 (D) 11/5

5. $\int \frac{e^{2x}(1+2x)}{sin^2(xe^{2x})} dx \ \ equals$

   (A) $-\cot(xe^{2x}) + c$ (B) $\cot(xe^{2x}) + c$ (C) $\tan(xe^{2x}) + c$ (D) $-\tan(xe^{2x}) + c$

6. If θ is the angle between any two vectors $\vec{a}$ $and\ \vec{b}, then\ \vec{a}.\vec{b} = -|\vec{a} \times \vec{b}|$, when θ is equal to

   (A) 0 (B) $\frac{\pi}{4}$ (c) $\frac{3\pi}{4}$ (D) $\frac{5\pi}{4}$

Section B

Question numbers 7 to 12 carry 2 marks each

7. By using elementary operations, find the matrix A, if inverse of matrix A= $\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$.
8. If $sin^2y + \cos(xy) = \pi, find\ \frac{dy}{dx}$
9. Find $\int sin5x\sqrt{1 + cos5x}\ dx$
10. Find the equation of a curve passing through the point (-2,3), given that the slope of tangent to the curve at any point(x,y) is $\frac{3x}{y}$.
11. Find the vector equation of the plane passing through the intersection of the planes. $\vec{r}.(2\hat{i}+2\hat{j}-3\hat{k})=7$, $\vec{r}.(2\hat{i}+5\hat{j}+3\hat{k})=9$ and through the point(2,1,3)

Or

Find the value of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{5p} = \frac{z-3}{2}$ *and* $\frac{7-7x}{3p} = \frac{y-7}{1} = \frac{6-z}{5}$ are at right angles.

12. Given that the events A and B are such that P(A)=$^1/_2$, P(AUB)=3/5 and P(B)=p. find p if A and B are independent events.

SECTION-C

Question numbers 13 to 25 carry 4 marks each

13. Consider f:$R_+ \to [-5, \infty)$ given by f(x)=$9x^2 + 6x - 5$. Show that f is one-one and onto. Hence find f^{-1}, where R_+ represents the set of all non-negative real numbers.

14. Find the values of x which satisfy the equation $sin^{-1}x + sin^{-1}(1-x) = cos^{-1}x$

15. If A=$\begin{bmatrix} p & -1 \\ q & 1 \end{bmatrix}, B = \begin{bmatrix} -1 & 1 \\ -2 & -1 \end{bmatrix}$ $and\ (A+B)^2 = A^2 + B^2 + 2AB,\ find\ p\ and\ q$

or

Using properties of the determinants, evaluate the following determinant

$$\begin{vmatrix} (\sqrt{13} + 2\sqrt{3}) & 2\sqrt{5} & \sqrt{5} \\ (\sqrt{15} + \sqrt{26}) & 5 & \sqrt{10} \\ (3 + \sqrt{65}) & \sqrt{15} & 5 \end{vmatrix}$$

16. If x=a sin2t(1+cos2t) and y=b cos2t(1-cos2t), show that $\left(\frac{dy}{dx}\right)_{at\ t=\frac{\pi}{4}} = \frac{b}{a}$

Or

If y=$e^{acos^{-1}x}, -1 \le x \le 1, show\ that\ (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$

17. Show that the normal at any point θ to the curve , x=acosθ+aθsinθ, y=asinθ-aθcosθ is at a constant distance from origin.

18. Find :$\int (6-5x)\sqrt{4+5x-3x^2}dx$

Or

Find : $\int \frac{x^2}{x^4+5x^2+4}dx.$

19. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{sin^2x}{sinx+cosx}dx.$

20. Find the particular solution of the differential equation
$\frac{dy}{dx} = 1 + x + y^2 + xy^2, given\ that\ y = 0\ when\ x = 0$

21. Find the general solution of the differential equation $(1 + tany)(dx - dy) + 2xdy = 0$

22. Show that $\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}\ \ are\ coplanar\ if\ \vec{a}, \vec{b}, \vec{c}\ are\ coplanar$

23. Find the foot of perpendicular from the point (2, 4, -1) to the line $\vec{r} = \left(-5\hat{\imath} - 3\hat{\jmath} + 6\hat{k}\right) + \mu\left(\hat{\imath} + 4\hat{\jmath} - 9\hat{k}\right)$. Also find this perpendicular distance.

24. Find the probability distribution of the maximum of the two scores obtained when a die is thrown twice. Also find the mean of the distribution.

25. There are three urns containing 2white and 3black balls, 3white and 2black balls, and 4white and 1black balls respectively. A ball is drawn at random from anyone of the urns and is found to be white. Find the probability that the ball was drawn from second urn.

Section-D

Question numbers 26 to 30 carry 6 marks each.

26. If A=$\begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, $find\ A^{-1}$ and use it to solve the following system of linear equations:

x-2y=10, 2x-y-z=8, -2y+z=7

27. Show that the height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle α, is one third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 tan^2\alpha$.

28. Using integration find the area bounded by the lines y=4x+5, y=5-x and 4y=x+5

29. If $\vec{a}, \vec{b}, \vec{c}\ are$ three non-zero vectors such that $\vec{a} \times \vec{b} = \vec{c}\ and\ \vec{b} \times \vec{c} = \vec{a}$,then show that $\vec{a}, \vec{b}, \vec{c}$ are mutually at right angles and $|\vec{b}| = 1\ and\ |\vec{c}| = |\vec{a}|$.

Or

Show that the lines $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z}{-1}\ and\ \frac{x-4}{3} = \frac{1-y}{2} = z - 1$ are coplanar. Also find the equation of the plane containing these lines.

30. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foods A and B are available at a cost of Rs.4 and Rs.3 per unit respectively. If one unit of a food A contains 200 units of vitamins, 1 unit of minerals and 40 units of calories while one unit of food B contains 100 units of vitamins, 2 units of minerals and 40 units of calories, find what combination of foods should be used to have the least cost? Make an L.P.P and solve graphically.

Marking scheme

Class-XII Maths

Section-A

Q.No.	Value points and solution	Marks
1-6	1.(C) 2.(D) 3.(A) 4.(C) 5.(A) 6.(C)	1×6=6
	Section-B	
7.	$\begin{pmatrix}1 & 2\\ 2 & 5\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}A^{-1}$; using $R_2 \to R_2-2R_1$ *we get* $\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}=\begin{pmatrix}1 & 0\\ -2 & 1\end{pmatrix}A^{-1}$; *Using* $R_1 \to R_1-2R_2$, *we get* $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=\begin{pmatrix}5 & -2\\ -2 & 5\end{pmatrix}A^{-1}$ => $A=\begin{pmatrix}5 & -2\\ -2 & 5\end{pmatrix}$	½ ½ ½ + ½
8.	$sin^2y+\cos(xy)=\pi \ldots\ldots(I)$ *Differentiating* (I), *we get* $2sinycosy\frac{dy}{dx}-\sin(xy)\left[x\frac{dy}{dx}+y\right]=0$ Or $\frac{dy}{dx}(sin2y-xsin(xy))=ysin(xy)$ => $\frac{dy}{dx}=\frac{ysin(xy)}{sin2y-xsin(xy)}$	1 1
9.	$I=\int sin5x\sqrt{1+cos5x}\,dx$ => $let\ 1+cos5x=t$ => $-5sin5xdx=dt$ => $sin5xdx=-\frac{1}{5}dt \therefore I=\int -\frac{1}{5}\sqrt{t}dt=-\frac{1}{5}.\frac{2}{3}(1+cos5x)^{\frac{3}{2}}+c$	1 1
10.	The slope of tangent is $\frac{dy}{dx}=\frac{3x}{y}$ => $ydy=3xdx$ Integrating we get $\frac{y^2}{2}=\frac{3x^2}{2}+c \ldots\ldots\ldots(i)$ (i) Passes through (-2,3) $\therefore \frac{9}{2}=\frac{3}{2}\times 4+c$ => $c=\frac{-3}{2}$ $\therefore$ *eqn. of curve is* $\frac{y^2}{2}=\frac{3}{2}x^2-\frac{3}{2}$ *or* $y^2=3x^2-3$	1 ½ ½
11.	The equation of plane passing through the intersection of two given planes is $[\vec{r}.(2\hat{\imath}+2\hat{\jmath}-3\hat{k})-7+\lambda[\vec{r}.(2\hat{\imath}+5\hat{\jmath}+3\hat{k})-9]=0]$ =>$\vec{r}.[(1+\lambda)2\hat{\imath}+(2+5\lambda)\hat{\jmath}+(-3+3\lambda)\hat{k}]-7-9\lambda=0$ ……….(i) (i) passes through the point (2,1,3), the vector $2\hat{\imath}+\hat{\jmath}+3\hat{k}$ should satisfy it => $(2\hat{\imath}+\hat{\jmath}+3\hat{k}).[(1+\lambda)2\hat{\imath}+(2+5\lambda)\hat{\jmath}+(-3+3\lambda)\hat{k}]-7-9\lambda=0$ Which gives $9\lambda=10$ *or* $\lambda=\frac{10}{9}$ $\therefore$ Required equation of plane is $\vec{r}.\left(\frac{38}{9}\hat{\imath}+\frac{68}{9}\hat{\jmath}+\frac{3}{9}\hat{k}\right)=17$ *or* $\vec{r}.(38\hat{\imath}+68\hat{\jmath}+3\hat{k})=153$	½ ½ ½ ½
	Or In standard form, the equation of the line can be written as $\frac{x-1}{-3}=\frac{y-2}{\frac{5}{7}p}=\frac{z-3}{2}$ & $\frac{x-1}{\frac{-3p}{7}}=\frac{y-7}{1}=\frac{z-6}{-5}$ The lines are perpendicular if $(-3)(\frac{-3p}{7})+\frac{5}{7}p(1)+2(-5)=0$ Or $\frac{9p}{7}+\frac{5p}{7}=10$ Or p=$\frac{70}{14}=5$	½ ½ ½ ½

12.	We know that P(AUB)=P(A)+P(B)-P(A∩B) Or $\frac{3}{5}=\frac{1}{2}+p-P(A\cap B) => P(A\cap B)=\frac{1}{2}+p-\frac{3}{5}=p-\frac{1}{10}$ As A and B are independent events P(A∩ B)=P(A).P(B) Or $p-\frac{1}{10}=\frac{1}{2}P \;=>\; p=\frac{1}{5}$	1 1
	Section-c	
13.	Let $x_1, x_2 \in \boldsymbol{R}_+$ such that $f(x_1)=f(x_2)$ $\therefore 9x_1^2+6x_1-5=9x_2^2+6x_2-5$ or $9[(x_1-x_2)(x_1+x_2)]+6(x_1-x_2)$=0 ⇨ $(x_1-x_2)[9x_1+9x_2+6]=0$ As $9x_1+9x_2+6\neq 0 \;=> x_1=x_2 \;=> f$ is one − one Let Y be any arbitrary element of **R**+, then $f(x)=y \;=>\; 9x^2+6x-5=y$ Or $(3x+1)^2-6=y$ Or $\left[\frac{\sqrt{y+6}-1}{3}\right]=x \rightarrow f$ is onto function $f^{-1}(y)=\left[\frac{\sqrt{y+6}-1}{3}\right]$ Or $f^{-1}(x)=\frac{-1+\sqrt{x+6}}{2}$	1 ½ 1 ½ 1
14.	$sin^{-1}x+sin^{-1}(1-x)=cos^{-1}x$ or $2sin^{-1}x+sin^{-1}(1-x)=\frac{\pi}{2}$ Or $2\,sin^{-1}x=cos^{-1}(1-x)$ ⇨ $\cos(2sin^{-1}x)=1-x$ or $1-2sin^2(sin^{-1}x)=1-x$ Or $1-2x^2=1-x \;=>\; x=0,\frac{1}{2}$	1 1 1 1
15.	$(A+B)^2=A^2+B^2+2AB \;=>\; AB=BA$ $\begin{pmatrix} p & -1 \\ q & 1 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ -2 & -1 \end{pmatrix}=\begin{pmatrix} -1 & 1 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} p & -1 \\ q & 1 \end{pmatrix}$ $=> \begin{pmatrix} -p+2 & p+1 \\ -q-2 & q-1 \end{pmatrix}=\begin{pmatrix} -p+q & 2 \\ -2p-q & +1 \end{pmatrix}$ P+1=2 => P=1 Q=2 OR $\Delta=\begin{vmatrix} \sqrt{13}+2\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{26} & 5 & \sqrt{10} \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix};=\begin{vmatrix} 2\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15} & 5 & \sqrt{10} \\ 3 & \sqrt{15} & 5 \end{vmatrix}+\begin{vmatrix} \sqrt{13} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{26} & 5 & \sqrt{10} \\ \sqrt{65} & \sqrt{15} & 5 \end{vmatrix}$	1 1 1 1 1 ½
	$\Delta=\sqrt{5}.\sqrt{3}\begin{vmatrix} 2 & 2 & \sqrt{5} \\ \sqrt{5} & \sqrt{5} & \sqrt{10} \\ \sqrt{3} & \sqrt{3} & 5 \end{vmatrix}+\sqrt{13}.\sqrt{5}\begin{vmatrix} 1 & 2\sqrt{5} & 1 \\ \sqrt{2} & 5 & \sqrt{2} \\ \sqrt{5} & \sqrt{15} & \sqrt{5} \end{vmatrix}$ =0+0=0	1 ½ 1

16.	$x = a\sin 2t(1+\cos 2t),\ \ y = b\cos 2t(1-\cos 2t)$ $= 2a\sin 2t.\cos^2 t, \qquad y = 2b\cos 2t.\sin^2 t$ $\frac{dx}{dt} = 2a[\cos^2 t.2\cos 2t + \sin 2t(-2\cos t\sin t)] = 2a[2\cos^2 t\cos 2t - \sin^2 2t]$ $\frac{dy}{dt} = 2b[\cos 2t.2\sin 2t + \sin^2 t(-2\sin 2t)] = b[\sin 4t - 4\sin 2t.\sin^2 t]$ $(\frac{dy}{dx})_{t=\pi/4} = \frac{(\frac{dy}{dt})t=\pi/4}{(\frac{dx}{dt})t=\pi/4} = \frac{b}{a}\left[\frac{-\frac{4.1.1}{2}}{-2}\right] = \frac{b}{a}$ **Or** $Y=e^{a\cos^{-1}x}, \frac{dy}{dx} = e^{a\cos^{-1}x}.\frac{(-a)}{\sqrt{1-x^2}} = \frac{-ay}{\sqrt{1-x^2}}$ $\sqrt{1-x^2}\frac{dy}{dx} + ay = 0: Diff. again\ we\ get$ $\sqrt{1-x^2}\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\frac{dy}{dx} + \frac{ady}{dx} = 0$ or $\sqrt{1-x^2}\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\frac{dy}{dx} - \frac{a^2y}{\sqrt{1-x^2}}=0$ Or $(1-x^2)\frac{d^2y}{dx} - x\frac{dy}{dx} - a^2y = 0$	1 1 1+1 1 ½ 1 ½ 1
17.	x=acosθ+aθsinθ, y=asinθ-aθcosθ $\frac{dx}{dQ} = -a\sin\theta + a(\theta\cos\theta + \sin\theta) = a\theta\cos\theta,$ $\frac{dy}{d\theta} = a\cos\theta - a(\cos\theta - \theta\sin\theta) = a\theta\sin\theta$ $\therefore \frac{dy}{dx} = \tan\theta \Rightarrow slope\ of\ normal = -\cot\theta = -\frac{\cos\theta}{\sin\theta}$ $\therefore Equation\ of\ normal\ is$ $y - (a\sin\theta - a\theta\cos\theta) = -\frac{\cos\theta}{\sin\theta}[x - (a\cos\theta + a\theta\sin\theta)]$ $y\sin\theta - a\sin^2\theta + a\theta\sin\theta\cos\theta = -x\cos\theta + a\cos^2\theta + a\theta\sin\theta\cos\theta$ xcosθ+ysinθ=a $\therefore Distance\ from\ origin = \left\|\frac{a}{\sin^2\theta+\cos^2\theta}\right\| = a = constant$	1 ½ ½ 1 1
18.	$6\text{-}5x = A(\frac{d}{dx}(4+5x-3x^2)) + B = A(5-6x) + B \Rightarrow A = \frac{5}{6},\ B = \frac{11}{6}$ $\therefore I = \frac{5}{6}\int(5-6x)\sqrt{4+5x-3x^2}dx + \frac{11}{6}\int\sqrt{4+5x-3x^2}$ $= \frac{5}{6}\times\frac{2}{3}(4+5x-3x^2)^{3/2} + \frac{11}{6}\int\sqrt{-3\left((x-\frac{5}{6})^2 + (\frac{\sqrt{73}}{6})^2\right)}$ $= \frac{5}{9}(4+5x-3x^2)^{3/2} + \frac{11}{6}\sqrt{3}\int\sqrt{(\frac{\sqrt{73}}{6})^2 - (x-\frac{5}{6})^2}\,dx$ $= \frac{5}{9}(4+5x-3x^2)^{3/2} + \frac{11}{2\sqrt{3}}\left[\frac{6x-5}{12}\sqrt{\frac{4}{3}+\frac{5}{3}x-x^2} + \frac{73}{72}\sin^{-1}\frac{6x-5}{\sqrt{73}}\right] + c$ Or $I = \int\frac{x^2}{x^4+5x^2+4}dx, \quad let\ x^2 = t \Rightarrow \quad \frac{x^2}{x^4+5x^2+4} = \frac{t}{(t+1)(t+4)}$ $= \frac{A}{(t+1)} + \frac{B}{(t+4)}$ Getting $A = -\frac{1}{3}, b = \frac{4}{3}$ $\therefore I = -\frac{1}{3}\int\frac{dx}{(x^2+1)} + \frac{4}{3}\int\frac{dx}{x^2+4} = -\frac{1}{3}\tan^{-1}x + \frac{2}{3}\tan^{-1}\frac{x}{2} + c$	1 1 1 1 1 1 1+1

19.	$I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx,$ *using properties of definite integrals, we get* $I = \int_0^{\frac{\pi}{2}} \frac{\sin^2(\frac{\pi}{2} - x)}{\sin(\frac{\pi}{2} - x) + \cos(\frac{\pi}{2} - x)} dx = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\sin x + \cos x} dx$ $\therefore 2I = \int_0^{\pi/2} \frac{dx}{\sin x + \cos x} = \frac{1}{\sqrt{2}} \int_0^{\pi/2} \frac{dx}{\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x} = \frac{1}{\sqrt{2}} \int_0^{\pi/2} \sec\left(x - \frac{\pi}{4}\right) dx$ $= \frac{1}{\sqrt{2}}\left[\log\left\|\sec\left(x - \frac{\pi}{4}\right) + \tan\left(x - \frac{\pi}{4}\right)\right\|\right]_0^{\pi/2}$ $= \frac{1}{\sqrt{2}}\left[\log\left\|\sec\frac{\pi}{4} + \tan\frac{\pi}{4}\right\| - \log\left\|\sec\left(-\frac{\pi}{4}\right) + \tan\left(-\frac{\pi}{4}\right)\right\|\right]$ $= \frac{1}{\sqrt{2}}\left[\log\frac{\sqrt{2}+1}{\sqrt{2}-1}\right] = \frac{1}{\sqrt{2}}\log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)\left(\frac{\sqrt{2}+1}{\sqrt{2}+1}\right) = \frac{1}{\sqrt{2}}\log(\sqrt{2}+1)^2 = \sqrt{2}\log(\sqrt{2}+1)$	1 1 1 1 1
20.	$\frac{dy}{dx} = (1+x)(1+y^2) \Rightarrow \frac{dy}{1+y^2} = (1+x)dx$ $\tan^{-1} y = x + \frac{x^2}{2} + c$ X=0, y=0 => c=0 => y=tan$(x+\frac{x^2}{2})$	1 1+1 1
21.	The given diff. Eqn. can be written as $\frac{dx}{dy} + \frac{2}{1+\tan y} x = 1$ I.f.=$e^{\int \frac{2\cos y}{\sin y + \cos y} dy} = e^{\int \frac{(\sin y + \cos y) + (\cos y - \sin y)}{\sin y + \cos y} dy}$ $= e^y(\sin y + \cos y)$ $\therefore$ *the solution is* $x.e^y(\sin y + \cos y) = \int e^y(\sin y + \cos y)dy + c$ $= e^y \sin y + c$ $[$*of the form* $\int e^x(f(x) + f'(x))]dx$	1 1 1
22.	$[(\vec{a} + \vec{b}) \times (\vec{b} + \vec{c})].(\vec{c} + \vec{a}) = [\vec{a} \times \vec{b} + \vec{a} \times \vec{c} + \vec{b} \times \vec{b} + \vec{b} \times \vec{c}].(\vec{c} + \vec{a})$ $= [\vec{a} \times \vec{b} + \vec{a} \times \vec{c} + \vec{b} \times \vec{c}].(\vec{c} + \vec{a})$ $=(\vec{a} \times \vec{b}).\vec{c} + (\vec{a} \times \vec{c}).\vec{c} + (\vec{b} \times \vec{c}).\vec{c} + (\vec{a} \times \vec{b}).\vec{a} + (\vec{a} \times \vec{c}).\vec{a} + (\vec{b} \times \vec{c}).\vec{a}$ $= (\vec{a} \times \vec{b}).\vec{c} + (\vec{b} \times \vec{c}).\vec{a} = 2[\vec{a}\vec{b}\vec{c}]$ We know that $\vec{a}, \vec{b}, \vec{c}$ are coplanar if $[\vec{a}, \vec{b}, \vec{c}] = 0$ $\therefore (\vec{a} + \vec{b}).(\vec{b} + \vec{c}).(\vec{c} + \vec{a})$ *are also coplaner*	1 ½ 1 ½ 1

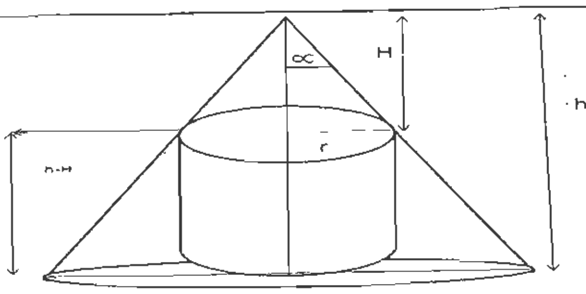
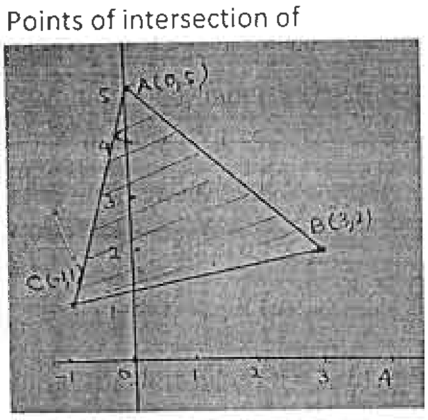
<table>
<tr><td>23.</td><td>The Cartesian form of line is $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}=\lambda$ ....(I)
A general point say P on the lines $\lambda-5, 4\lambda-3, -9\lambda+6$
$\therefore d.r's\ of\ PQ\ are\ \lambda-7, 4\lambda-7, -9\lambda+7$
PQ is perpendicular to (I), so
$(\lambda-7)+4(4\lambda-7)-9(-9\lambda+7)=0$
$\lambda-7+16\lambda-28+81\lambda-63=0$
$98\lambda=98 \Rightarrow \lambda=1$
$\therefore$ *the point*, P on (I) is (-4, 1, -3)
$\therefore$ *the distance PQ is* $\sqrt{(2+4)^2+(4-1)^2+(-1+3)^2}$
$=\sqrt{36+9+4}=7$</td><td>1
½
½

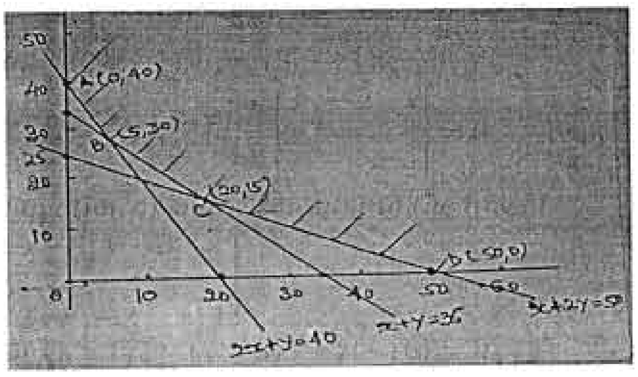
1

1</td></tr>
<tr><td>24.</td><td>Let X: Maximum of two scores
$\therefore$ X   1   2   3   4   5   6
$P(x)$ $1/36$ $3/36$ $5/36$ $7/36$ $9/36$ $11/36$
$xP(x)$ $1/36$ $6/36$ $15/36$ $28/36$ $45/36$ $66/36$
Mean=$\sum xP(x)=\frac{161}{36}$</td><td>½
1½
1

1</td></tr>
<tr><td>25.</td><td>Let E_1, E_2 *and* E_3 be the events of selecting Urns I, II, and III respectively. A be the event of taking out white ball from an urn
$P(E_1)=P(E_2)=P(E_3)=\frac{1}{3}$
$P(A/E_1)=\frac{2}{5}, P(A/E_2)=\frac{3}{5}, P(A/E_3)=\frac{4}{5}$
$\therefore\ P(E_2/A)=\frac{\frac{1}{3}\cdot\frac{3}{5}}{\frac{1}{3}\left[\frac{2}{5}+\frac{3}{5}+\frac{4}{5}\right]}=\frac{3/5}{9/5}=\frac{1}{3}$</td><td>½
1½

1+1</td></tr>
<tr><td></td><td><u>Section-D</u></td><td></td></tr>
<tr><td>26.</td><td>$\lvert A\rvert=1(-1-2)-2(-2)=1\neq0 \Rightarrow A^{-1}\ exists, A=\begin{pmatrix}1&2&0\\-2&-1&-2\\0&-1&1\end{pmatrix}$
$adj\ A=\begin{pmatrix}-3&-2&-4\\2&1&2\\2&1&3\end{pmatrix}, A^{-1}=\begin{pmatrix}-3&-2&-4\\2&1&2\\2&1&3\end{pmatrix}$
The given system can be written as $A^T x=B$, *when* $B=\begin{pmatrix}10\\8\\7\end{pmatrix}$
$\begin{pmatrix}x\\y\\z\end{pmatrix}=x=(A^{-1})^T B=\begin{pmatrix}-3&2&2\\-2&1&1\\-4&2&3\end{pmatrix}\begin{pmatrix}10\\8\\7\end{pmatrix}=\begin{pmatrix}0\\-5\\-3\end{pmatrix}$
$\therefore$ x=0, y=-5, z=-3</td><td>1
1½ + ½
½
½
2</td></tr>
</table>

27.		1
	$\frac{r}{H} = tan \propto => \quad H = r cot \propto$	
	V= volume of cylinder $= \pi r^2(h - r cot \propto)$	
	$= \pi r^2 h - \pi r^3 cot \propto$	1
	$\frac{dv}{dr} = 2\pi rh - 3\pi r^2 cot \propto$	1
	$\frac{d^2v}{dr^2} = 2\pi h - 6\,\pi r cot \propto$	1
	$\frac{dv}{dr} = 0 \quad => r = \frac{2}{3} h\ tan \propto$	
	$showing\ that\ when\ r = \frac{2}{3} htan \propto, \frac{d^2v}{dr^2} < 0 \ => Maximum$	½
	$h - H = h - \frac{2}{3}h = \frac{h}{3}$	1
	$\therefore Maximum\ volume\ of\ cylinder = \pi\left(\frac{4}{9}h^2 tan^2 \propto\right).\frac{h}{3} = \frac{4}{27}\pi h^3 tan^2 \propto$	½
28.	i) 4x+5=y, ii) y=5-x, iii)4y=x+5	
	Points of intersection of	
		
	(i) and (ii) is A(0,5) (ii) and (iii), is B(3,2), (iii) and (i) is C(-1,1)	1 ½
	$\therefore$ Reqd area= $\int_{-1}^{0}(4x+5)dx + \int_{0}^{3}(5-x)dx - \int_{-1}^{3}\frac{x+5}{4}dx$	1 ½
	$= (2x^2+5x)_{-1}^{0} + (5x - \frac{x^2}{2})_0^3 - \frac{1}{4}(\frac{x^2}{2} + 5x)_{-1}^{3}$	1 ½
	$= 3 + \frac{21}{2} - 6 = \frac{15}{2}\ sq\ units$	1 ½
29.	(i) $\left.\begin{matrix}\vec{a}\times\vec{b} = \vec{c} => \ \vec{a}\perp\vec{c}\ and\ \vec{b}\perp\vec{c} \\ \vec{b}\times\vec{c} = \vec{a} => \vec{b}\perp\vec{a}\ \text{and}\ \ \vec{c}\perp\vec{a}\end{matrix}\right\} => \ \vec{a}\perp\vec{b}\perp\vec{c}$	1
	From (i), $\lvert\vec{a}\rvert\lvert\vec{b}\rvert sin\frac{\pi}{2} = \lvert\vec{c}\rvert and\ from\ (ii) \quad \lvert\vec{b}\rvert\lvert\vec{c}\rvert sin\frac{\pi}{2} = \lvert\vec{a}\rvert$	1
	⇨ $\lvert\vec{a}\rvert\lvert\vec{b}\rvert = \lvert\vec{c}\rvert \quad \dots\dots\dots\dots(i)$	1
	$\lvert\vec{b}\rvert\lvert\vec{c}\rvert = \lvert\vec{a}\rvert \quad \dots\dots(ii)$	1
	$\lvert\vec{b}\rvert\lvert\vec{a}\rvert\lvert\vec{b}\rvert = \lvert\vec{a}\rvert \quad \Rightarrow \quad \lvert\vec{b}\rvert^2 = 1 \quad or \quad \lvert\vec{b}\rvert = 1 \quad (ii)$	1
	$\lvert\vec{a}\rvert.1 = \lvert\vec{c}\rvert \quad \Rightarrow \quad \lvert\vec{a}\rvert = \lvert\vec{c}\rvert[from\ (i) and (ii)]$	1

	OR	
	Lines are coplanar if $(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = 0$	1
	$\vec{a}_1 = (1,3,0),\ \vec{a}_2 = (4,1,1) \ => \ \vec{a}_2 - \vec{a}_1 = (3,-2,1)$	1
	$\vec{b}_1 = (2,4,-1), \vec{b}_2 = (3,-2,1)$	½
	$\therefore\ \vec{a}_2 - \vec{a}_1 . \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} 3 & -2 & 1 \\ 2 & 4 & -1 \\ 3 & -2 & 1 \end{vmatrix} = 0$ Hence the lines are coplanar	1 1
	Equation of plane containing the lines is	
	$\begin{vmatrix} x-1 & y-3 & z \\ 2 & 4 & -1 \\ 3 & -2 & 1 \end{vmatrix} = 0$	½
	$or\ (x-1)2 - (y-3)5 + z(-16) = 0$	½
	$2x - 5y - 16z + 13 = 0$	½
30.	Total cost function if x and y units of A and B respectively consumed is c=4x+3y Under the constraints 2x+y≥,40, x+2y≥,50, x+y≥35,x,y≥0	2
		2
	$C_A = 120$ $C_B = 110$ $C_C = 125$ $C_D = 200$	
	Cost is minimum at B $\therefore$ 5 *units of food A*} $\therefore$ 30 *units of food B be used*}	2